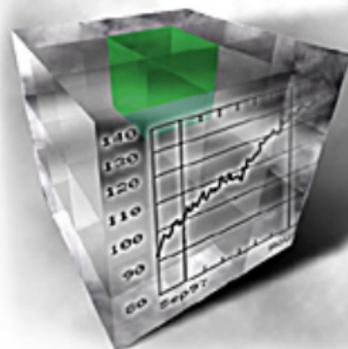


# Datawarehouse and OLAP

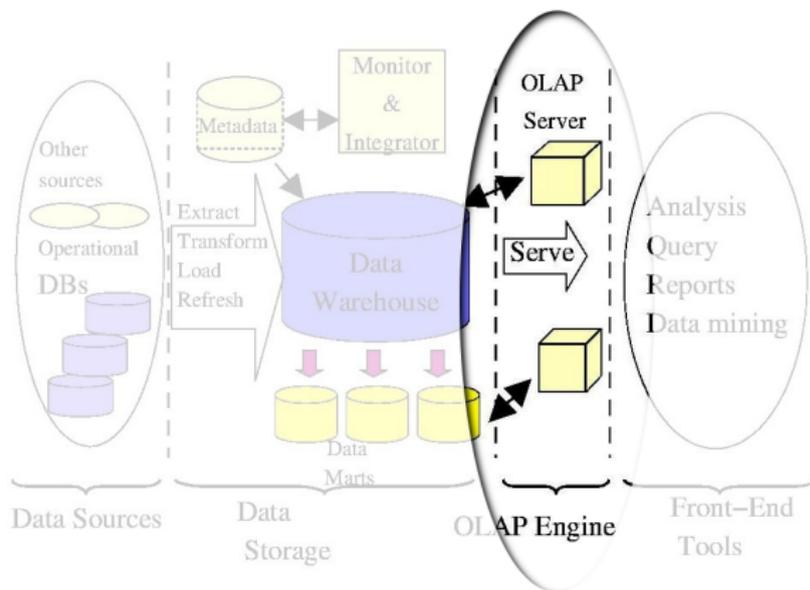
OLAP



## Syllabus, materials, notes, etc.

See <http://www.info.univ-tours.fr/~marcel/dw.html>

# On-Line Analytical Processing



today

OLAP models and languages

industry standards

formal models

## logical models and query languages

no commonly agreed formal logical model and query language

- ▶ standards for data and metadata exchange
- ▶ query languages
  - ▶ SQL extensions
  - ▶ MDX: the de facto standard?
- ▶ many research works

# SQL Extensions

## SQL extensions

- ▶ Microsoft MDX
- ▶ ANSI SQL 99

## microsoft's MDX

cf. SQL Server on-line doc

<http://msdn.microsoft.com/en-us/library/ms145506.aspx>  
(10/2009)

typical instruction

```
SELECT < axis_specification >    [, < axis_specification > ...]  
FROM   < cube_specification >  
WHERE  < slicer_specification >
```

## syntax: select-from-where?

| clause | parameter           | gives                       |
|--------|---------------------|-----------------------------|
| SELECT | 1 relation per axis | axes of resulting cross-tab |
| FROM   | 1 cube name         | the queried cells           |
| WHERE  | 1 tuple             | the queried slice           |

## syntax: typical functions

|               |                               |   |
|---------------|-------------------------------|---|
| navigation    | PARENT<br>CHILDREN<br>MEMBERS | a member's parent<br>a member's children<br>a level's members<br>or a dimension's members |
| structuration | CROSSJOIN                     | dimension nesting   |
| ranking       | TOPCOUNT                      | first members   |

## example

SalesCube with six dimensions:

- ▶ SalesPerson
- ▶ Geography (Countries > Regions > States > Cities)
- ▶ Quarters (Quarters > Months > Days)
- ▶ Years
- ▶ Measures (Sales, PercentChange, and BudgetedSales)
- ▶ Products (Category > Product)

## example

```
SELECT  CROSSJOIN({[Venkatrao], [Netz]},
                {[USA_North].CHILDREN, [USA_South], [Japan]})
        ON COLUMNS,
        {[Qtr1].CHILDREN, [Qtr2], [Qtr3], [Qtr4].CHILDREN}
        ON ROWS
FROM    [SalesCube]
WHERE   ([Sales], [1991], [Products].[All])
```

{ } delimit sets, [ ] delimit terms,

. identifies terms

## example

Sales for 1991, All Products

|      |     | Venkatrao |        |           |       | Netz      |        |           |       |
|------|-----|-----------|--------|-----------|-------|-----------|--------|-----------|-------|
|      |     | USA       |        |           | Japan | USA       |        |           | Japan |
|      |     | USA_North |        | USA_South |       | USA_North |        | USA_South |       |
|      |     | Seattle   | Boston |           |       | Seattle   | Boston |           |       |
| Qtr1 | Jan | 00        | 10     | 20        | 30    | 40        | 50     | 60        | 70    |
|      | Feb | 01        | 11     | 21        | 31    | 41        | 51     | 61        | 71    |
|      | Mar | 02        | 12     | 22        | 32    | 42        | 52     | 62        | 72    |
| Qtr2 |     | 03        | 13     | 23        | 33    | 43        | 53     | 63        | 73    |
| Qtr3 |     | 04        | 14     | 24        | 34    | 44        | 54     | 64        | 74    |
| Qtr4 | Oct | 05        | 15     | 25        | 35    | 45        | 55     | 65        | 75    |
|      | Nov | 06        | 16     | 26        | 36    | 46        | 56     | 66        | 76    |
|      | Dec | 07        | 17     | 27        | 37    | 47        | 57     | 67        | 77    |

language closure?

## semantics

```
SELECT  CROSSJOIN({[Venkatrao], [Netz]},  
                {[USA_North].CHILDREN, [USA_South], [Japan]})  
        ON COLUMNS,  
        {[Qtr1].CHILDREN, [Qtr2], [Qtr3], [Qtr4].CHILDREN}  
        ON ROWS  
FROM    [SalesCube]  
WHERE   ([Sales], [1991], [Products].[All])
```

1. SELECT, WHERE : cross product of queries over the dimension tables to compute the desired references
2. FROM : semi join with the data cube of the fact table to obtain the measures' value

## formally

if  $C$  is an  $n$ -dimensional cube with

- ▶ dimension tables  $D_1, \dots, D_n$
- ▶ and fact table  $F$  of schema  $F[D_1, \dots, D_n, val]$

an MDX query  $q$  with  $k$  axes is a tuple  $\langle q_1, \dots, q_k, q_{k+1} \rangle$  where

- ▶  $\forall i, i' \neq i \in [1, k + 1], \text{sort}(q_i) \neq \text{sort}(q_{i'})$
- ▶  $\forall i \in [1, k], q_i$  is a relational query  $q_i^1(D_i^1) \times \dots \times q_i^x(D_i^{x_i})$   
where  $\{D_i^1, \dots, D_i^{x_i}\} \subseteq \{D_1, \dots, D_n\}$  are the dimensions  
appearing on axis  $i$
- ▶  $q_{k+1}$  is a conjunctive query yielding exactly one tuple  
 $q_{k+1}^1(D_{k+1}^1) \times \dots \times q_{k+1}^y(D_{k+1}^y)$  where  
 $\{D_{k+1}^1, \dots, D_{k+1}^y\} \subseteq \{D_1, \dots, D_n\}$  are the dimensions  
appearing in the WHERE clause

## formally

Let  $\{D_a^1, \dots, D_a^z\} \subseteq \{D_1, \dots, D_n\}$  be the dimension tables not appearing in the MDX query, then  $q_a^i(D_a^i)$  extracts the 'all' member of dimension  $D_a^i$

an MDX query  $q$  with  $k$  axes on an  $n$ -dimensional cube corresponds to the set of cells

$$q_1^1(D_1^1) \times \dots \times q_{k+1}^y(D_{k+1}^y) \times q_a^1(D_a^1) \times \dots \times q_a^z(D_a^z) \times (\gamma_{D_1; \text{agg}(\text{val})}(F) \cup \gamma_{D_1, D_2; \text{agg}(\text{val})}(F) \cup \dots \cup F)$$

where the  $\{D_1^1, \dots, D_a^z\} = \{D_1, \dots, D_n\}$ , and  $\gamma$  is the grouping/aggregation operator

## example

Suppose the following star schema for SalesCube:

- ▶ SalesPerson[all\_salesperson,name]
- ▶ Geography[all\_geography,countries,regions,states,cities]
- ▶ Quarters[all\_quarters,quarters,months,days]
- ▶ Years[all\_years,years]
- ▶ Measures[name]
- ▶ Products[all\_products, category, product]

sales[SalesPerson,Geography,Quarters,Years,Measures,Product,value]

## example

*[Geography].MEMBERS* translates

```
select distinct all_geography from Geography union  
select distinct countries from Geography union  
select distinct regions from Geography union  
select distinct states from Geography union  
select distinct cities from Geography ;
```

## example

`{[Qtr1].CHILDREN,[Qtr2],[Qtr3],[Qtr4].CHILDREN}`

translates

```
select distinct months from Quarters where quarters='Qtr1' union  
select distinct quarters from Quarters where quarters='Qtr2' union  
select distinct quarters from Quarters where quarters='Qtr3' union  
select distinct months from Quarters where quarters='Qtr4';
```

## example

```
CROSSJOIN({[Venkatrao], [Netz]},  
{[USA_North].CHILDREN, [USA_South], [Japan]})
```

translates

```
select name, T.location  
from SalesPerson, (  
select distinct states as location from Geography  
where regions='USA_North'  
union  
select distinct 'USA_South' as location from Geography  
union  
select distinct 'Japan' as location from Geography  
) T  
where name='Netz' or name='Venkatrao'
```

## example

WHERE ([Sales], [1991], [Products].[All]) translates

```
select measure, years, all_product  
from Measures, Years, Products  
where name='Sales' and years=1991 and all_products='All'
```

## example

```
SELECT [Qtr1].CHILDREN ON ROWS  
FROM SalesCube  
WHERE ([Sales],[1991])
```

translates into the star join query over the fact table sales

```
select years,months,name,sum(value)  
from Years, Quarters, Measures, sales  
where quarters='Qtr1' and years=1991 and name='Sales'  
and Years.years=sales.Years and Quarters.days=sales.Quarters and  
Measures.name=sales.Measures  
group by years,months,name
```

## example

```
SELECT [Qtr1].CHILDREN ON ROWS  
FROM SalesCube  
WHERE ([Sales],[1991])
```

suppose now the data cube is stored in a table of schema

```
salesDataCube[SalesPerson,Geography,Quarters,Years,Measures,Product,value]
```

then the MDX query translates

```
select value from salesDataCube natural right outer join ( select years as Years,  
months as Quarters, name as Measures from Years, Quarters, Measures where  
quarters='Qtr1' and years=1991 and name='Sales' ) S where  
SalesPerson='All' and Geography='All' and Product='All';
```

## calculated members / ad-hoc aggregates

```
WITH MEMBER [Measures].[Special Discount] AS
    [Measures].[Sales] * 0.75
SELECT [Measures].[Special Discount] ON COLUMNS,
    [Products].[Product].MEMBERS ON ROWS
FROM [SalesCube]

WITH MEMBER [Product].[All Products].[Drink].[Avg Drinks]
AS
    'AVG([Product].[All Products].[Drink].Children,
    [Measures].[Sales])'
SELECT { [Product].[All Products].[Drink].Children,
    [Product].[All Products].[Drink].[Avg Drinks] } ON COLUMNS,
    [USA].Children ON ROWS
FROM [SalesCube]
WHERE [Measures].[Sales]
```

## ANSI SQL-99

adds OLAP features to SQL-92 :

- ▶ GROUPING SETS: extends GROUP BY
- ▶ CUBE, ROLLUP: particular cases of GROUPING SETS
- ▶ ranking: extends ORDER BY
- ▶ windowing: moving averages or sums

supported by Oracle, IBM DB2, SAS, partially by MySQL...

## examples: consider the facts

```
SELECT jour, ville, SUM(ventes)
FROM c1
GROUP BY jour,ville
```

| c <sub>1</sub> | jour              | ville              | ventes          |
|----------------|-------------------|--------------------|-----------------|
|                | jour <sub>1</sub> | ville <sub>1</sub> | v <sub>11</sub> |
|                | jour <sub>1</sub> | ville <sub>2</sub> | v <sub>12</sub> |
|                | jour <sub>2</sub> | ville <sub>1</sub> | v <sub>21</sub> |
|                | ⋮                 | ⋮                  | ⋮               |
|                | jour <sub>q</sub> | ville <sub>p</sub> | v <sub>qp</sub> |

# CUBE

computes the UNION of GROUP BY for every subset of the set of attributes

```
SELECT    jour, ville, SUM(ventes)
FROM      c1
GROUP BY  CUBE(jour,ville)
```

yields the groupings

$$\{(jour, ville), (jour), (ville), \emptyset\}$$

# CUBE

| jour              | ville              | ventes               |
|-------------------|--------------------|----------------------|
| jour <sub>1</sub> | ville <sub>1</sub> | v <sub>11</sub>      |
| jour <sub>1</sub> | ville <sub>2</sub> | v <sub>12</sub>      |
| jour <sub>1</sub> | NULL               | v <sub>1_ALL</sub>   |
| jour <sub>2</sub> | ville <sub>1</sub> | v <sub>21</sub>      |
| ⋮                 | ⋮                  | ⋮                    |
| NULL              | ville <sub>1</sub> | v <sub>ALL_1</sub>   |
| ⋮                 | ⋮                  | ⋮                    |
| NULL              | ville <sub>p</sub> | v <sub>ALL_p</sub>   |
| NULL              | NULL               | v <sub>ALL_ALL</sub> |

# ROLLUP

computes the UNION of GROUP BY of each prefix of the set of attributes

```
SELECT    jour, ville, SUM(ventes)
FROM      c1
GROUP BY  ROLLUP(jour,ville)
```

yields the groupings

$$\{(jour, ville), (jour), \emptyset\}$$

# ROLLUP

| jour              | ville              | ventes               |
|-------------------|--------------------|----------------------|
| jour <sub>1</sub> | ville <sub>1</sub> | v <sub>11</sub>      |
| jour <sub>1</sub> | ville <sub>2</sub> | v <sub>12</sub>      |
| jour <sub>1</sub> | NULL               | v <sub>1-ALL</sub>   |
| jour <sub>2</sub> | ville <sub>1</sub> | v <sub>21</sub>      |
| ⋮                 | ⋮                  | ⋮                    |
| NULL              | NULL               | v <sub>ALL-ALL</sub> |

# ROLLUP

```
SELECT    jour, ville, SUM(ventes)
FROM      c1
GROUP BY  ROLLUP(jour), ROLLUP(ville)
```

yields the groupings

$$\{(jour), \emptyset\} \times \{(ville), \emptyset\} \equiv \{(jour, ville), (jour), (ville), \emptyset\}$$

## ROLLUP with hierarchy

```
SELECT    jour, mois, années, SUM(ventes)
FROM      c1, dimension_time
WHERE     c1.jour=dimension_time.jour
GROUP BY  ROLLUP(années,mois,jour)
```

computes the aggregates for all levels of the time dimension:  
jour → mois → année

# GROUPING SETS

consider the facts

| $c_1$ | jour              | ville              | pièce              | ventes           |
|-------|-------------------|--------------------|--------------------|------------------|
|       | jour <sub>1</sub> | ville <sub>1</sub> | pièce <sub>1</sub> | v <sub>111</sub> |
|       | jour <sub>1</sub> | ville <sub>2</sub> | pièce <sub>1</sub> | v <sub>121</sub> |
|       | jour <sub>2</sub> | ville <sub>1</sub> | pièce <sub>2</sub> | v <sub>212</sub> |
|       | ⋮                 | ⋮                  | ⋮                  | ⋮                |
|       | jour <sub>q</sub> | ville <sub>p</sub> | pièce <sub>r</sub> | v <sub>qpr</sub> |

# GROUPING SETS

multiple GROUP BY precising the desired UNION

attribute nesting allows to separate simple GROUP BY from UNION of GROUP BY

CUBE and ROLLUP are particular cases of GROUPING SETS

# GROUPING SETS

GROUP BY  
GROUPING SETS  
((jour, ville, pièce))

≡ GROUP BY jour, ville, pièce

GROUP BY  
GROUPING SETS  
(jour, ville, pièce)

≡ GROUP BY jour  
UNION  
GROUP BY ville  
UNION  
GROUP BY pièce

GROUP BY  
GROUPING SETS  
(jour,(ville,pièce))

≡ GROUP BY jour  
UNION  
GROUP BY ville, pièce

# ranking

ranks an ORDER BY result

```
SELECT jour,ville,rank() OVER (ORDER BY sum(ventes) DESC)
FROM c1
```

“top-n” query

```
SELECT jour,ville,rank() OVER (ORDER BY sum(ventes) DESC) as rang
FROM c1
ORDER BY rang
FETCH FIRST 5 ROWS ONLY
```

# windowing

cumulative or moving aggregates

```
SELECT ville,jour,avg(ventes) OVER (ORDER BY ville, jour ROWS  
BETWEEN 1 PRECEDING AND 1 FOLLOWING)  
FROM c1
```

# Standards and API

## Standards and API

- ▶ CWM
- ▶ XMLA
- ▶ java API

## Common Warehouse Metamodel

[www.omg.org/cwm](http://www.omg.org/cwm)

- ▶ standard for BI and DW metadata exchange
- ▶ based upon
  - ▶ UML, Unified Modeling Language, an OMG modeling standard
  - ▶ MOF, Meta Object Facility, an OMG metamodeling and metadata repository standard
  - ▶ XMI, XML Metadata Interchange, an OMG metadata interchange standard
- ▶ supported by IBM, SAS, Oracle, Hyperion, Pentaho (mondrian), ...

## XML for analysis

[www.xmla.org](http://www.xmla.org)

- ▶ specification for a set of XML message interfaces
  - ▶ data access interaction between a client application and an analytical data provider
  - ▶ over the Internet
- ▶ based upon XML, MDX
- ▶ supported by Microsoft, Hyperion, SAP, SAS, Pentaho, ...

## the late JOLAP API

### J2EE API

- ▶ proposed by IBM, Sun, Hyperion, Oracle, Nokia, SAS, ...
- ▶ final draft september 2003
- ▶ approval june 2004
- ▶ nothing since then

*JOLAP was not properly implemented by any vendors and has been quietly forgotten. [...] we do not expect JOLAP to be resurrected.*

OLAP report, 2007

## olap4j API

[www.olap4j.org](http://www.olap4j.org)

- ▶ common java API for any OLAP server
- ▶ JDBC for OLAP (extension to JDBC)
- ▶ based upon XMLA, MDX, AJAX
- ▶ open source project, supported by Pentaho, ...

# Formal models and languages

## OLAP query languages

### 4 examples

- ▶ Gyssens and Lakshmanan, VLDB'97 (algebra)
- ▶ Agrawal, Gupta, Sarawagi, ICDE'97 (algebra)
- ▶ Hacid, Marcel et Rigotti, DOOD'97 (datalog)
- ▶ Vassiliadis, Skiadopoulos, CAISE'00 (algebra)

## introductory remark

few logical/physical optimisations

expressiveness poorly characterised

close to the relational model

## Gyssens and Lakshmanan's algebra : intuitions

cube =  
    content  
        set of relations  
+ structure  
    member/measure status

11 operators exploitant la séparation contenu/structure

## data model

content of an  $n$ -dimensional cube  $c$  of dimensions  $d_1, \dots, d_n$

- ▶ a set of  $n$  relations,  $r_{d_1}, \dots, r_{d_n}$ 
  - ▶ each tuple with an unique id
  - ▶ each tuple corresponds to a member
- ▶ a relation  $r_m$ 
  - ▶ has each key attribute of  $r_{d_1}, \dots, r_{d_n}$
  - ▶ has more attributes for the measures

## schema

schema of a cube  $\langle D, R, par \rangle$

- ▶  $D = \{d_1, \dots, d_n\}$  a set of dimensions
- ▶  $R = \{A_1, \dots, A_m\}$  a set of d'attributes
- ▶  $par : D \rightarrow 2^{\{A_1, \dots, A_m\}}$  with
  - ▶  $\forall i, j = 1, \dots, n, i \neq j, par(d_i) \cap par(d_j) = \emptyset$
  - ▶  $\cup_{d \in D} par(d) \subseteq R$
- ▶  $M = R - \cup_{1 \leq i \leq n} par(d_i)$

## instance

instance of an  $n$ -dimensional cube of schema  $\langle D, R, par \rangle$  :

$rd_1(T_{id}, par(d_1)), \dots, rd_n(T_{id}, par(d_n)),$

$r_m(rd_1.T_{id}, \dots, rd_n.T_{id}, M)$

- ▶  $\pi_{T_{id}}(r_{d_1}) \times \dots \times \pi_{T_{id}}(r_{d_n}) = \pi_{r_{d_1}.T_{id}, \dots, r_{d_n}.T_{id}}(r_m)$
- ▶  $\forall i = 1, \dots, n, T_{id}$  is a primary key of  $r_{d_i}$
- ▶  $\forall i, j = 1, \dots, n, i \neq j, \pi_{T_{id}}(r_{d_i}) \cap \pi_{T_{id}}(r_{d_j}) = \emptyset$

## correspondance

for a cube  $\tau$  of schema  $S = \langle D, R, par \rangle$ ,  $rep(\tau)$

- ▶ relational representation of  $\tau$
- ▶ a relation  $r$  of schema  $R$

for a relation  $r$  of schema  $R$ ,  $tab_S(r)$

- ▶ cube representation of  $r$
- ▶ a cube of schema  $S = \langle D, R, par \rangle$

## contents of the cube dimensions

|         |          |        |
|---------|----------|--------|
| produit | $T_{id}$ | pièces |
|         | p1       | écrous |
|         | p2       | clous  |
|         | p3       | vis    |

|      |          |         |
|------|----------|---------|
| lieu | $T_{id}$ | régions |
|      | l1       | est     |
|      | l2       | sud     |
|      | l3       | nord    |
| l4   | ouest    |         |

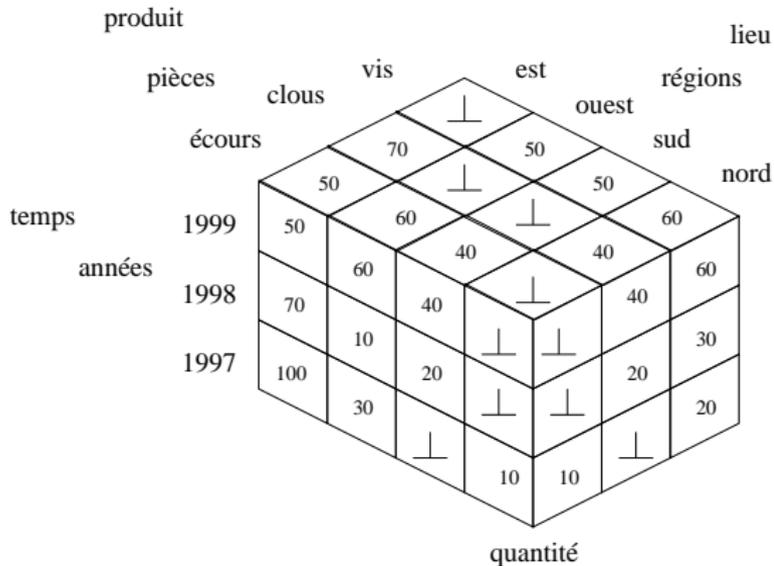
|       |          |        |
|-------|----------|--------|
| temps | $T_{id}$ | années |
|       | t1       | 1997   |
|       | t2       | 1998   |
|       | t3       | 1999   |

## contents of the cube cells

| $r_m$ | $temps.T_{id}$ | $produit.T_{id}$ | $lieu.T_{id}$ | quantité |
|-------|----------------|------------------|---------------|----------|
|       | t1             | p1               | l1            | 60       |
|       | t1             | p1               | l2            | 40       |
|       | ⋮              | ⋮                | ⋮             | ⋮        |
|       | t2             | p2               | l3            | 20       |
|       | ⋮              | ⋮                | ⋮             | ⋮        |

## structure

one possible representation of  $r_{d_1}, \dots, r_{d_n}, r_m$   
 ventes



## set operations

7 operators for manipulating contents

$\tau_1$  et  $\tau_2$  of schema  $S_1 = \langle D_1, R_1, par_1 \rangle$

$\tau$  of schema  $S = \langle D, R, par \rangle$

with  $D_1 \cap D = \emptyset$  et  $R_1 \cap R = \emptyset$

- ▶ unary operators (SPR):  $op(\tau) = tab_S(op(rep(\tau)))$
- ▶ cross product (C):  $\tau_1 \times \tau = tab_{S'}(rep(\tau_1) \times rep(\tau))$  with  $S' = \langle D_1 \cup D, R_1 \cup R, par_1 \cup par \rangle$
- ▶ binary (UID):  $\tau_1 op \tau_2 = tab_{S_1}(rep(\tau_1) op rep(\tau_2))$

## restructuring

2 operators on the cube's structure

$\tau$  a cube of schema  $\langle D, R, par \rangle$

- ▶  $unfold_X^d(\tau)$  is a cube of schema  $\langle D \cup \{d\}, R, par' \rangle$ 
  - ▶  $\forall d_i \in D, par'(d_i) = par(d_i)$
  - ▶  $par'(d) = X$
- ▶  $fold^d(\tau)$  is a cube of schema  $\langle D - \{d\}, R, par' \rangle$ 
  - ▶  $\forall d_i \in D - \{d\}, par'(d_i) = par(d_i)$

## granularity

not part of the model  
deals with only the contents

2 operators using external functions

- ▶ classification: for grouping tuples
- ▶ consolidation: for aggregating tuples

## classification

$\tau$  a cube of schema  $\langle D, R, par \rangle$

$K(\tau, f) = tab_{S'}(K(rep(\tau), f))$  where

- ▶  $\{A_1, \dots, A_k\} \subset R$
- ▶  $f : dom(f.A_1) \times \dots \times dom(f.A_k) \rightarrow 2^{dom(A_1) \times \dots \times dom(A_k)}$
- ▶  $S' = \langle D, R \cup \{f.A_1, \dots, f.A_k\}, par' \rangle$
- ▶  $f.A_i \in par'(d) \iff A_i \in par(d)$

## classification

$K(r, f)$  is a relation of schema

$(f.A_1, \dots, f.A_k, A_1, \dots, A_k, \dots, A_m)$

and of instance

$\{(a_1, \dots, a_k, a'_1, \dots, a'_k, \dots, a'_m) \mid$   
 $(a'_1, \dots, a'_k) \in f(a_1, \dots, a_k) \wedge (a'_1, \dots, a'_k, \dots, a'_m) \in r\}$

## consolidation

$\tau$  is a cube of schema  $\langle D, R, par \rangle$

$A(\tau, g) = tab_{S'}(A(rep(\tau), g))$  where

- ▶  $\{A_1, \dots, A_k\} \subset R = \{A_1, \dots, A_m\}$
- ▶  $B$  a new attribute  $B \notin \{A_1, \dots, A_k\}$
- ▶  $A_j, k + 1 \leq j \leq m$  of same type than  $B$
- ▶  $S' = \langle D, \{A_1, \dots, A_k, B\}, par \rangle$

## consolidation

$$g_{A_j \rightarrow B} : 2^{\text{dom}(A_{k+1}) \times \dots \times \text{dom}(A_m)} \rightarrow \text{dom}(B)$$

$A(r, g)$  is a relation of schema  $\{A_1, \dots, A_k, B\}$

and of instance

$$\{(a_1, \dots, a_k, b) \mid b = g(\{(a_{k+1}, \dots, a_m) \mid (a_1, \dots, a_k, a_{k+1}, \dots, a_m) \in r\})\}$$

## example of a typical query formulation

$unfold^{lignes}_{années, produits} ($   
 $fold^{produits} ($   
 $fold^{années} ($   
 $\pi_{années, produits, quantité}(sud)))$  where sud is defined by

$\sigma_{régions=sud} ($   
 $\pi_{années, régions, produits, quantité} ($   
 $\sigma_{années=a+1 \wedge quantité > q \wedge régions=r \wedge produits=p} ($   
 $\rho_{quantités, régions, produits, années \rightarrow q, r, p, a}(ventes) \times ventes)))$

## example of a typical query formulation

$A(K(\text{ventes}, f_{gliss}), g_{\text{quantité} \rightarrow \text{moyenne}})$ , with

$$f_{gliss}(x) = \{x' \mid x' = x \vee x' = x + 1\}$$

$$g_{\text{quantité} \rightarrow \text{moyenne}}(S) = (1/|S|) \sum_{(w,x,y,z) \in S} (z)$$

## algebra of Agrawal, Gupta, Sarawagi

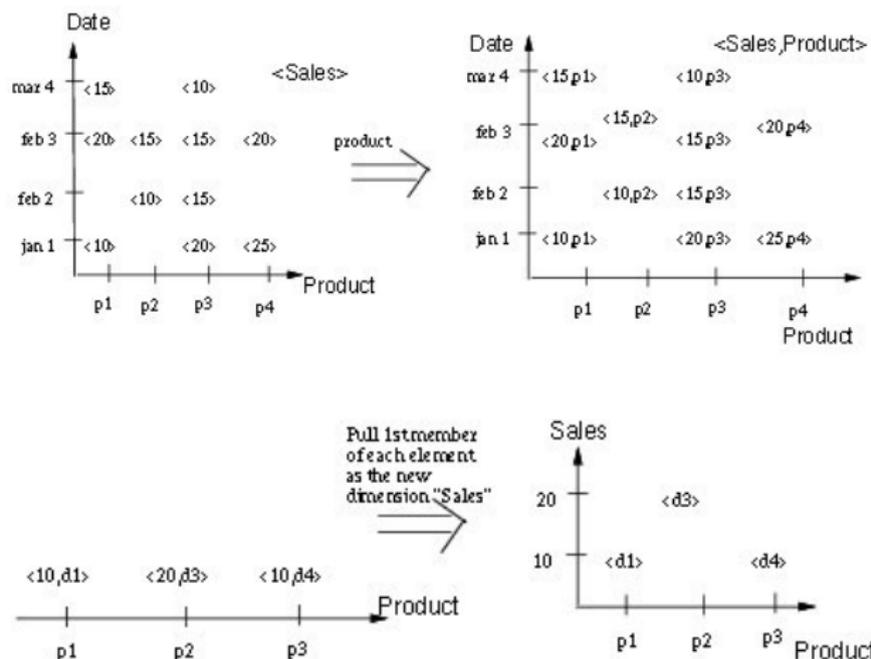
model: an  $n$ -dimensional cube  $C$

- ▶  $D_{i, i \in [1, n]}$  dimensions of domain  $dom_{D_i}$
- ▶  $f$  a function from  $dom_{D_1} \times \dots \times dom_{D_n}$  to
  - ▶ 1
  - ▶ 0
  - ▶ a tuple with arity  $< n$

# operators

- ▶ push, pull
- ▶ projection, selection, join
- ▶ aggregation

## push and pull



## push

consider  $C_1 = (D_1, \dots, D_n, f_1)$  and  $C_2 = (D_1, \dots, D_n, f_2)$

$push(C_1, D_i) = C_2$  with

$f_2(d_1, \dots, d_n) =$

- ▶ 0 if  $f_1(d_1, \dots, d_n) = 0$
- ▶  $(d_i)$  if  $f_1(d_1, \dots, d_n) = 1$
- ▶ the tuple  $(t_1, \dots, t_m, d_i)$  if  $f_1(d_1, \dots, d_n) = (t_1, \dots, t_m)$

# pull

consider  $C_1 = (D_1, \dots, D_{n-1}, f_1)$  and  $C_2 = (D_1, \dots, D_n, f_2)$

the measures of  $C_1$  cannot be 0 or 1

$pull(C_1, D_n, i) = C_2$  with

$f_2(d_1, \dots, d_n) =$

- ▶  $(m_1, \dots, m_{i-1}, m_{i+1}, \dots, m_k)$  si  
 $f_1(d_1, \dots, d_{n-1}) = (m_1, \dots, m_i, \dots, m_k)$
- ▶ 0 otherwise

## projection

consider  $C_1 = (D_1, \dots, D_n, f_1)$  with  $|dom_{D_i}| = 1$

$remove(C_1, D_i) = C_2$  with

- ▶  $C_2 = (D_1, \dots, D_{i-1}, D_{i+1}, \dots, D_n, f_2)$
- ▶  $f_2(d_1, \dots, d_{i-1}, d_{i+1}, \dots, d_n) = f_1(d_1, \dots, d_{i-1}, d_i, d_{i+1}, \dots, d_n)$

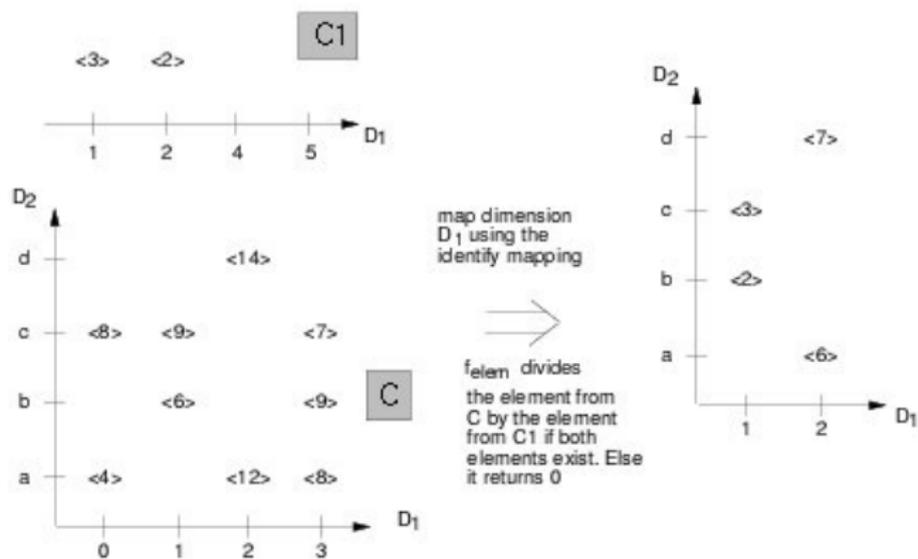
## selection

consider  $C_1 = (D_1, \dots, D_n, f_1)$

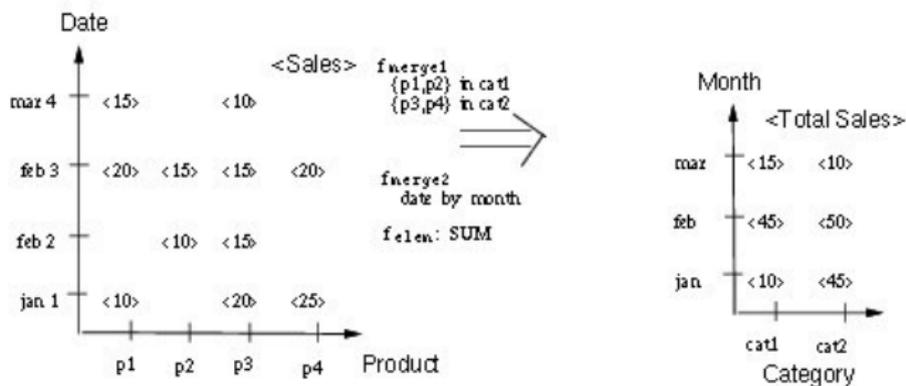
$\sigma_{D_i, P}(C_1) = C_2$  with

- ▶  $C_2 = (D_1, \dots, D_n, f_2)$
- ▶  $dom'_{D_i} = P(dom_{D_i})$
- ▶  $f_2$  is the restriction of  $f_1$  to  
 $dom_{D_1} \times \dots \times dom'_{D_i} \times \dots \times dom_{D_n}$

# join



## aggregation



## datalog: intuitions

*une référence de cellule = un atome datalog*

|                       |  |
|-----------------------|--|
| monovaluation         | dépendances fonctionnelles                                     |
| granularité           | décrite par une relation spécifique<br>(extension + intention) |
| flexibilité du schema | syntaxe d'ordre supérieur                                      |

## modèle de données

|              |  |
|--------------|--|
| nom atomique | écrous   |
| nom composé  | écrous . 1999  |
| référence    | ventes(écrous,1999,ouest)  |
| cellule      | ventes(écrous,1999,ouest) :⟨50⟩  |
| cube         | {ventes(écrous,1999,ouest) :⟨50⟩,<br>:<br>ventes(clous,1998,nord) :⟨20⟩} |

## monovaluation

$\{ \dots, \text{ventes}(\text{écrous}, 1999, \text{ouest}) : \langle 50 \rangle, \dots, \text{ventes}(\text{écrous}, 1999, \text{ouest}) : \langle 70 \rangle, \dots \}$

est interdit, donc

$a(b,c) : d \leftarrow .$

$a(b,c) : e \leftarrow .$

est interdit aussi

## termes du langage

$\mathcal{D}$  ensemble de noms atomiques,  $\mathcal{V}$  ensemble de variables

|                   |      |  |
|-------------------|------|--|
| <i>atomicName</i> | $:=$ | $c \in \mathcal{D} \mid v \in \mathcal{V}$             |
| <i>name</i>       | $:=$ | <i>atomicName</i> $\mid$ <i>name.name</i>              |
| <i>contents</i>   | $:=$ | $\langle \textit{name}, \dots, \textit{name} \rangle$  |
| <i>reference</i>  | $:=$ | <i>name</i> ( <i>name</i> , ..., <i>name</i> )         |
| <i>cellAtom</i>   | $:=$ | <i>reference</i> : <i>contents</i>                     |
| <i>atom</i>       | $:=$ | <i>cellAtom</i> $\mid$ <i>groupingAtom</i>             |
| <i>literal</i>    | $:=$ | <i>atom</i> $\mid$ <i>aggregateSubgoal</i>             |
| <i>body</i>       | $:=$ | <i>literal</i> , ..., <i>literal</i> $\mid$ $\epsilon$ |
| <i>head</i>       | $:=$ | <i>atom</i>  |
| <i>rule</i>       | $:=$ | <i>head</i> $\leftarrow$ <i>body</i>                   |

## termes du langage : granularité

$\mathcal{AGG}$  ensemble d'opérateurs d'agrégat

$groupingAtom \quad := \quad in(atomicName, atomicName)$

$aggregateSubgoal \quad := \quad atomicName = f(reference)$

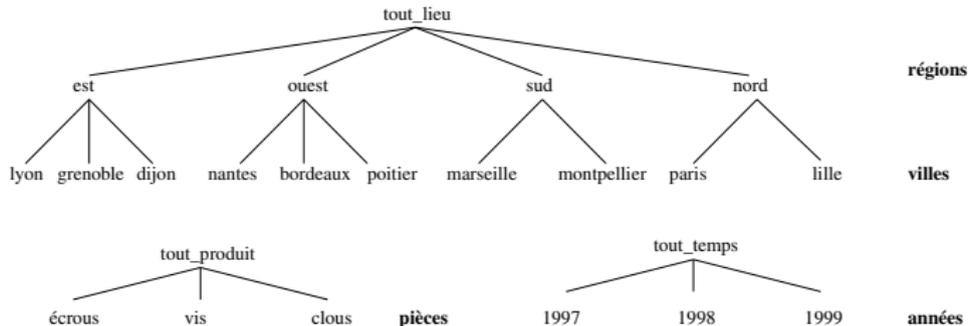
$literal \quad := \quad atom \mid aggregateSubgoal$

où  $f \in \mathcal{AGG}$ .

## termes du langage : granularité

description des groupements

{in(vis,tout\_produit), in(1997,tout\_temps), ... in(lyon,est),  
in(est,tout\_lieu)}



## restructurations

*split*

$$\text{ventes.R(P,A) :}\langle Q \rangle \leftarrow \text{ventes(P,A,R) :}\langle Q \rangle.$$

## restructurations

*split*

$\text{ventes.R(P,A) : \langle Q \rangle} \leftarrow \text{ventes(P,A,R) : \langle Q \rangle}.$

*nest*

$\text{ventesNest(P.R, A) : \langle Q \rangle} \leftarrow \text{ventes(P,A,R) : \langle Q \rangle}.$

## restructurations

*split*

$$\text{ventes.R}(P,A) : \langle Q \rangle \leftarrow \text{ventes}(P,A,R) : \langle Q \rangle.$$

*nest*

$$\text{ventesNest}(P.R, A) : \langle Q \rangle \leftarrow \text{ventes}(P,A,R) : \langle Q \rangle.$$

*push*

$$\text{ventesPush}(P,R) : \langle A_1, Q_1, A_2, Q_2, A_3, Q_3 \rangle \leftarrow \text{ventes}(P,A_1,R) : \langle Q_1 \rangle,$$

$$\text{ventes}(P,A_2,R) : \langle Q_2 \rangle,$$

$$\text{ventes}(P,A_3,R) : \langle Q_3 \rangle,$$

$$A_1 < A_2,$$

$$A_2 < A_3.$$

## roll-up

$20 = \text{sum}(\text{ventes}(\text{vis}, \text{tout\_temps}, \text{est}))$

$\{\text{ventes}(\text{vis}, 1998, \text{est}): \langle 10 \rangle, \text{ventes}(\text{vis}, 1997, \text{est}): \langle 10 \rangle\}$

## roll-up

$20 = \text{sum}(\text{ventes}(\text{vis}, \text{tout\_temps}, \text{est}))$

$\{\text{ventes}(\text{vis}, 1998, \text{est}) : \langle 10 \rangle, \text{ventes}(\text{vis}, 1997, \text{est}) : \langle 10 \rangle\}$

roll-up sur la dimension temps

$\text{ventes}(P, \text{tout\_temps}, R) : \langle T \rangle \leftarrow T = \text{sum}(\text{ventes}(P, \text{tout\_temps}, R)),$   
in(P, tout\_produit),  
in(R, tout\_lieu).

## ordre des références

$J$  une interpretation à partir d'un ensemble de faits initial  $I$ .

$$\forall x, y \in \mathcal{D}, in_J(x, y) \iff in(x, y) \in J$$

$$\forall rf = n(n_1, \dots, n_p), rf' = n(n'_1, \dots, n'_p), rf <_J rf' \iff$$

- ▶  $rf \neq rf'$
- ▶  $\forall i \in [1, \dots, p],$ 
  - ▶  $in_J(n_i, n'_i)$
  - ▶ ou  $n_i = n'_i$

## sémantique des sous-buts d'agrégats

$B = k = f(n(n_1, \dots, n_p))$  un sous-but d'agrégat

$$detailRef_I^J(B) = \{ref(A) \mid A \in I, ref(A) <_J ref(B)\}$$

$$detailCont_I^J(B) = \{k \mid A \in I, ref(A) \in detailRef_I^J(B), k = val(A)\}$$

$B$  satisfait si

- ▶  $detailRef_I^J(B) \neq \emptyset$
- ▶  $f(detailCont_I^J(B))$  est défini
- ▶  $f(detailCont_I^J(B)) = k$

## caractéristiques

cadre formel proche du cadre datalog classique

- ▶ sémantique déclarative (type théorie des modèles)
- ▶ sémantique opérationnelle (type pppf) équivalente

programmes traduisibles en datalog

## exemple de spécification de requête typique

$R(A_2.P,ventes):\langle Q_2 \rangle \leftarrow ventes(P,A_2,R):\langle Q_2 \rangle,$

$ventes(P,A_1,R):\langle Q_1 \rangle,$

$Q_2 > Q_1,$

$A_2$  is  $A_1 + 1.$

## exemple de spécification de requête typique

$\text{resultat}(A.A',P,R):\langle M \rangle \leftarrow \text{ventes}(P,A,R):\langle S1 \rangle,$

$A'$  is  $A + 1,$

$\text{ventes}(P,A',R):\langle S2 \rangle,$

$M$  is  $(S1 + S2)/2.$

## exemple de spécification de requête typique

$\text{ventes}(P, A_2, R) : \langle Q_2 \rangle \leftarrow \text{ventes}(P, A_1, R) : \langle Q_1 \rangle,$

$A_2$  is  $A_1 + 1,$

$Q_2$  is  $Q_1 + Q_1/10,$

$A_2 \leq 2002,$

$A_1 \geq 1999.$

$\{ \text{ventes}(\text{clous}, 2000, \text{est}) : \langle 77 \rangle, \text{ventes}(\text{clous}, 2000, \text{nord}) : \langle 44 \rangle,$   
 $\text{ventes}(\text{vis}, 2000, \text{sud}) : \langle 55 \rangle, \text{ventes}(\text{clous}, 2001, \text{est}) : \langle 84.7 \rangle,$   
 $\text{ventes}(\text{clous}, 2001, \text{nord}) : \langle 48.4 \rangle, \dots \}$

## Groupements ad-hoc

| responsables | est  | ouest | sud  | nord |
|--------------|------|-------|------|------|
| vis          | john | mike  | bob  | mike |
| clous        | bob  | john  | bob  | mike |
| écrous       | john | bob   | john | bob  |

$\text{in}(P,N) \leftarrow \text{responsables}(P,R) : \langle N \rangle.$

$\text{in}(R,N) \leftarrow \text{responsables}(P,R) : \langle N \rangle.$

## Groupements ad-hoc

$\text{ventesResp}(R,P,A,N) : \langle Q \rangle \leftarrow \text{ventes}(P,A,R) : \langle Q \rangle,$   
 $\text{responsables}(P,R) : \langle N \rangle.$

$\text{résultat99}(\text{total},N) : \langle S \rangle \leftarrow S = \text{sum}(\text{ventesResp}(N,N,1999,N)),$   
 $\text{responsables}(-,-) : \langle N \rangle.$

|                   |      |     |      |
|-------------------|------|-----|------|
| <i>résultat99</i> | john | bob | mike |
| total             | 140  | 120 | 150  |

## Opérateur “data cube”

$\text{in}(\text{jour}_1, \text{mois}_1), \text{in}(\text{jour}_2, \text{mois}_1), \dots, \text{in}(\text{mois}_1, \text{année}_1), \dots,$   
 $\text{in}(\text{vendeur}_1, \text{ville}_1), \dots, \text{in}(\text{ville}_1, \text{pays}_1), \dots$

$\text{niveau}(X) \leftarrow \text{in}(X, Y)$

$\text{niveau}(Y) \leftarrow \text{in}(X, Y)$

$c(T, C): \langle S \rangle \leftarrow S = \text{sum}(c(T, C)), \text{niveau}(T), \text{niveau}(C).$

## Vassiliadis and Skiadopoulos's algebra

- ▶ extends former proposals
- ▶ a cube is a view over an underlying data set
- ▶ a dimension is a lattice
- ▶ the history of performed selections is kept for subsequent optimisations

# model

## data model

- ▶ data sets
- ▶ dimensions
- ▶ cubes

## algebraic operators

- ▶ navigation (roll-up and drill-down)
- ▶ selection
- ▶ split measure (projection for measures)

## data set

| Day       | Title              | Salesman | Store      | Sales |
|-----------|--------------------|----------|------------|-------|
| 6-Feb-97  | Symposium          | Netz     | Paris      | 7     |
| 18-Feb-97 | Karamazof brothers | Netz     | Seattle    | 5     |
| 11-May-97 | Ace of Spades      | Netz     | LosAngeles | 20    |
| 3-Sep-97  | Zarathustra        | Netz     | Nagasaki   | 50    |
| 3-Sep-97  | Report to El Greco | Netz     | Nagasaki   | 30    |
| 1-Jul-97  | Ace of Spades      | Venk     | Athens     | 13    |
| 1-Jul-97  | Piece of Mind      | Venk     | Athens     | 34    |
| :         |                    |          |            |       |

global data source *DS*

## dimensions and hierarchies

a dimension  $D$  is a lattice  $D = (L, <)$  with

- ▶  $L$  a set of levels  $L = (L_1, \dots, L_n, ALL)$
- ▶  $<$  a partial order over  $L$  such that,  $\forall i \in [1, n], L_1 < L_i < ALL$
- ▶ a family of functions  $anc_{L_1}^{L_2}$  that associates each element of  $dom(L_1)$  with one element of  $dom(L_2)$

## example

dimension  $Time = (L, <)$  with

- ▶  $L = \{Day, Month, Year, ALL\}$
- ▶  $Day < Month < Year < ALL$
- ▶  $dom(Day) = \{18 Feb 97, \dots\}$
- ▶  $dom(Month) = \{Feb 97, \dots\}$
- ▶  $dom(Year) = \{97, \dots\}$
- ▶  $dom(ALL) = \{all\}$
- ▶  $anc_{Day}^{Month}$ ,  $anc_{Month}^{Year}$ ,  $anc_{Year}^{ALL}$ ,

and for example:  $anc_{Day}^{Month}(18 Feb 97) = Feb 97$ ,  
 $anc_{Month}^{Year}(Feb 97) = 97$ ,  $anc_{Year}^{ALL}(97) = all$

## cube

a cube  $c$  of schema  $[L_1, \dots, L_n, M_1, \dots, M_m]$  is  
 $(DS_0, \varphi, [L_1, \dots, L_n, M_1, \dots, M_m], [agg_1(M_1^0), \dots, agg_m(M_m^0)])$

where

- ▶  $DS_0$  is a source data set of schema  $[L_1^0, \dots, L_n^0, M_1^0, \dots, M_k^0], k > m$
- ▶  $\varphi$  is a detailed selection condition
- ▶ the  $L_i$  are levels
- ▶  $M_1, \dots, M_m$  are aggregated measures
- ▶ les  $agg_i$  are aggregated functions

## example

let *sales\_97\_by\_Store* be a cube of schema [*Year*,*ALL*,*ALL*,*Store*,*result*]

(*DS*, *Year* = 97, [*Year*,*ALL*,*ALL*,*Store*,*result*], [*sum*(*Sales*)])

with

- ▶ *DS* is the data source of schema [*Day*,*Title*,*Salesman*,*Store*,*Sales*]
- ▶ *Year* = 97 is a selection condition on *DS*
- ▶ *result* corresponds to *sum*(*Sales*)

## algebra

let  $c^a$  be the initial cube

$$c^a = (DS_0, \varphi^a, [L_1^a, \dots, L_n^a, M_1^a, \dots, M_m^a], [agg_1^a(M_1^0), \dots, agg_m^a(M_m^0)])$$

navigation

$$\begin{aligned} navi(c^a, [L_1, \dots, L_n, M_1, \dots, M_m], agg_1, \dots, agg_m) = \\ (DS_0, \varphi^a, [L_1, \dots, L_n, M_1, \dots, M_m], [agg_1(M_1^0), \dots, agg_m(M_m^0)]) \end{aligned}$$

selection

$$\begin{aligned} \sigma_\varphi(c^a) = \\ (DS_0, \varphi^a \wedge \varphi, [L_1^a, \dots, L_n^a, M_1^a, \dots, M_m^a], [agg_1^a(M_1^0), \dots, agg_m^a(M_m^0)]) \end{aligned}$$

split measure

$$\begin{aligned} \pi_{M_m}(c^a) = \\ (DS_0, \varphi^a, [L_1^a, \dots, L_n^a, M_1^a, \dots, M_{m-1}^a], [agg_1^a(M_1^0), \dots, agg_m^a(M_{m-1}^0)]) \end{aligned}$$

## example

```
let sales_97_by_store =  
(DS, Year = 97, [Year, ALL, ALL, Store, result], [sum(Sales)])
```

## example

```
let sales_97_by_store =  
(DS, Year = 97, [Year, ALL, ALL, Store, result], [sum(Sales)])
```

```
navi(sales_97_by_store, [Year, ALL, ALL, ALL, result_year], sum) =
```

```
(DS, Year = 97, [Year, ALL, ALL, ALL, result_year], [sum(Sales)])
```

## example

let *sales\_97\_by\_store* =  
(*DS*, *Year* = 97, [*Year*, *ALL*, *ALL*, *Store*, *result*], [*sum*(*Sales*)])

*navi*(*sales\_97\_by\_store*, [*Year*, *ALL*, *ALL*, *ALL*, *result\_year*], *sum*) =

(*DS*, *Year* = 97, [*Year*, *ALL*, *ALL*, *ALL*, *result\_year*], [*sum*(*Sales*)])

$\sigma_{Store=Paris}$ (*sales\_97\_by\_store*) =

(*DS*, *Year* = 97  $\wedge$  *Store* =  
*Paris*, [*Year*, *ALL*, *ALL*, *Store*, *result*], [*sum*(*Sales*)])

## computation of a cube

1. apply the selection condition on source data  
each occurrence of  $L$  in  $\varphi$  is replaced with  $anc_{L^0}^L(L^0)$
2. replace the values of the levels for the tuples of the result with their respective ancestor values at the levels of the schema of the cube
3. group them into a single value for each measure, through the application of the appropriate aggregate function

## OLAP analysis

the cube  $c_2$  is obtained from the cube  $c_1$

- ▶ can  $c_2$  be computed from  $c_1$ ?
- ▶ can  $c_1$ 's tuples be used?

variation of the view subsumption problem

are there new problems due to hierarchies?

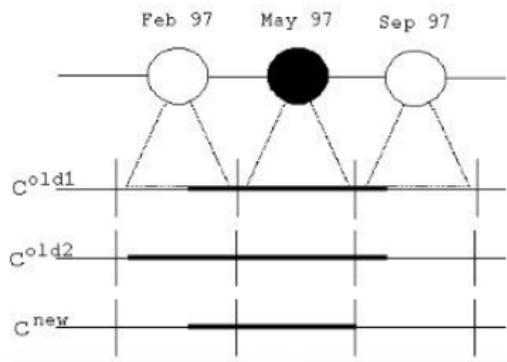
## OLAP analysis

$c_2$  must

- ▶ be defined on the same dimensions as  $c_1$ , at a greater or equal level
- ▶ be defined on the same measures of  $DS_0$  and with the same aggregation functions
- ▶ have a more restrictive selection condition over the same tuples as  $DS_0$

## example

$$c^i = [DS, \varphi_i, [Month, ALL, ALL, ALL, Sales], sum(sales)]$$



$$\varphi_{old1} = 18\text{-Feb-97} \leq Day \leq 3\text{-Sep-97} \wedge Salesman = Netz$$

$$\varphi_{old2} = 6\text{-Feb-97} \leq Day \leq 3\text{-Sep-97} \wedge Salesman = Netz$$

$$\varphi_{new} = 18\text{-Feb-97} \leq Day \leq 31\text{-May-97} \wedge Salesman = Netz$$

## test of the selection condition

partition the values w.r.t.  $c_2$ 's schema  
for each partition

test if there exists a partition for  $c_1$   
if yes

test if the 2 partitions comply

## before concluding...

note that we have only talk about query languages

now, what is an OLAP analysis?

short answer: a sequence of OLAP queries

is there more?

## conclusion

So far: we know quite a lot now

Next: what's next?